Math 32 Exam 2 2014 (Spring)

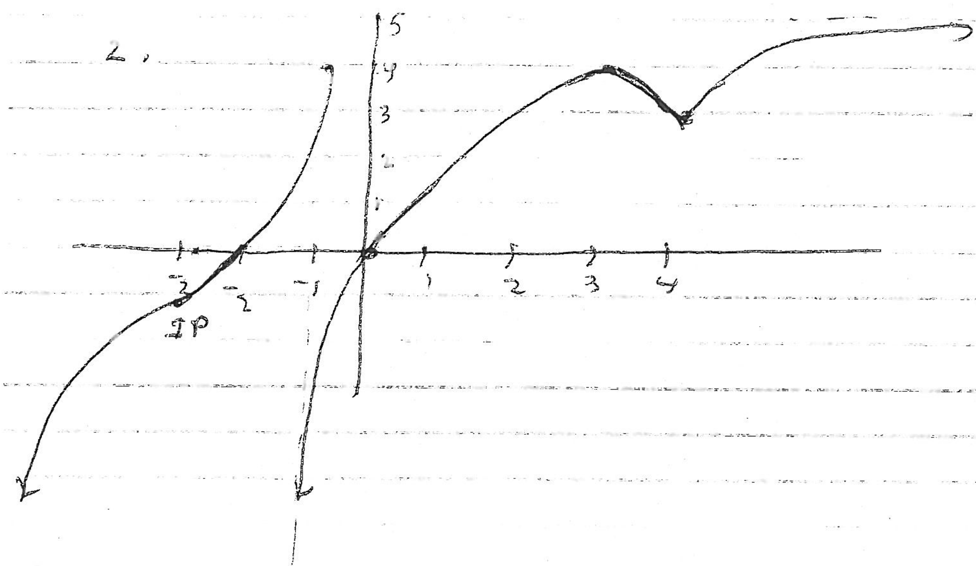
1. A) $(0, \infty)$ B) 1 C) None D) (e^{-3}, ∞) E) e^{-3}
F) None G) $(e^{-3/2}, \infty)$ H) $(0, e^{-3/2})$
I) $e^{-3/2}$

J) $\lim_{x\to 0^+} x^{1/3} \ln x \overset{0\cdot(-\infty)}{=} \lim_{x\to 0^+} \dfrac{\ln x}{x^{-1/3}}$

$\overset{-\infty/\infty}{=} \lim_{x\to 0^+} \dfrac{1/x}{-\frac{1}{3}x^{-4/3}} = \lim_{x\to 0^+} -3x^{1/3} = 0$

So there is <u>no</u> VA at $x=0$

2.



3. a) $y = \ln\left(\dfrac{e^x x^{2/3}}{\sqrt{1+x^4}}\right) = x + \frac{2}{3}\ln x - \frac{1}{2}\ln(1+x^4)$

$\dfrac{dy}{dx} = 1 + \dfrac{2}{3x} - \dfrac{1}{\cancel{2}} \dfrac{\cancel{4}2x^3}{1+x^4}$

b) $\ln y = \ln x \, \ln(2e^x + \sin x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \ln x\left[\dfrac{2e^x + \cos x}{2e^x + \sin x}\right] + \dfrac{1}{x}\ln(2e^x + \sin x)$

$\dfrac{dy}{dx} = (2e^x + \sin x)^{\ln x}\left\{\ln x\left[\dfrac{2e^x + \cos x}{2e^x + \sin x}\right] + \dfrac{\ln(2e^x + \sin x)}{x}\right\}$

c) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - \left(\frac{\cos^2 x}{2}\right)^2}} \cdot \dfrac{2\cos x(-\sin x)}{2}$

4. a) $\lim\limits_{x\to e} \dfrac{-1+\ln x}{\sin(x-e)} \overset{0/0}{=} \lim\limits_{x\to e} \dfrac{1/x}{\cos(x-e)} = \dfrac{1/e}{1} = \boxed{\dfrac{1}{e}}$

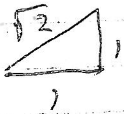
b) $\lim\limits_{x\to(\pi/4)^-} (\tan x)^{(1/(-x+\pi/4))}$ $\quad (1^\infty)$

Let $y = (\tan x)^{(1/(-x+\pi/4))}$

$$\ln y = \frac{\ln(\tan x)}{-x+\pi/4}$$

$$\lim_{x\to\frac{\pi}{4}^-} \ln y = \lim_{x\to\frac{\pi}{4}^-} \frac{\ln(\tan x)}{-x+\pi/4} \quad \frac{0}{0}$$

$$= \lim_{x\to\frac{\pi}{4}^-} \frac{\frac{\sec^2 x}{\tan x}}{-1} = \frac{2}{-1} = -2$$



Answer: $\boxed{e^{-2}}$

5. a) A continuous function on a closed interval has an absolute max and an absolute min.

b) $g(x) = x - \sqrt{x}$ on $[0,4]$, g is cont.

$g'(x) = 1 - \dfrac{1}{2\sqrt{x}}$ on $(0,4)$

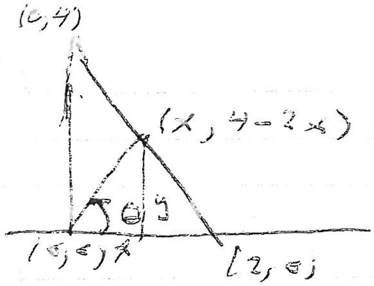
$= 0 \implies 1 = \dfrac{1}{2\sqrt{x}} \implies \sqrt{x} = \dfrac{1}{2}$

$\implies x = \dfrac{1}{4}$ is only C.P.

$g(0) = 0,\quad g(4) = 2,\quad g\left(\frac{1}{4}\right) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$\boxed{\text{Abs. max is } 2}$ $\boxed{\text{at } x = 4}$ $\quad$ $\boxed{\text{Abs min is } -\frac{1}{4}}$ $\boxed{\text{at } x = +\frac{1}{4}}$

c)



Given: $\frac{d\theta}{dt} = -\frac{1}{5}$ rad/min

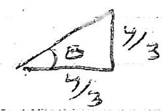
Find $\left.\frac{dx}{dt}\right|_{x=4/3 \text{ yd}}$

$\tan\theta = \frac{y}{x} = \frac{4-2x}{x} = \frac{4}{x} - 2$

$\sec^2\theta \, \frac{d\theta}{dt} = -\frac{4}{x^2}\frac{dx}{dt}$

If $x = \frac{4}{3}$, $y = 4 - 2\cdot\frac{4}{3} = \frac{4}{3}$

So



$\Rightarrow \left.\frac{dx}{d\theta}\right|_{x=4/3} = \left.\sec^2\theta\,\frac{d\theta}{dt}\cdot\frac{x^2}{(-4)}\right|_{x=4/3} = (\sqrt{2})^2\left(-\frac{1}{5}\right)\frac{\left(\frac{4}{3}\right)^2}{(-4)}$

$= 2\,\frac{1}{\cancel{4}\cdot 5}\left(\frac{\cancel{16}^{\,4}}{9}\right) = \boxed{\frac{8}{45}\text{ yd/min}}$

7 a) $\int\left(\sin x + 3e^x + \frac{1}{x}\right)dx$

$= \boxed{-\cos x + 3e^x + \ln|x| + C}$

b) $h'(x) = -3\sqrt{x} + 6\sec x\tan x$, $h(0) = 5$

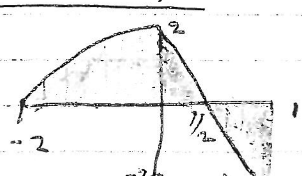
$h(x) = \int\left(-3\sqrt{x} + 6\sec x\tan x\right)dx$

$= -\cancel{3}\frac{2}{\cancel{3}}x^{3/2} + 6\sec x + C$

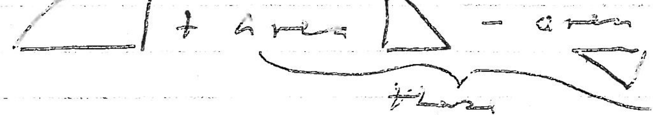
$5 = h(0) = 0 + 6 + C \Rightarrow C = -1$

So $\boxed{h(x) = -2x^{3/2} + 6\sec x - 1}$

c) $f(x) = \begin{cases}\sqrt{4-x^2}, & -2 \le x \le 0\\ 2-4x, & 0 < x \le 1\end{cases}$ i)



$\int_{-2}^{1} f(x)\,dx$ = area



+ area − area

these cancel

$= \frac{1}{4}\pi(2)^2 = \boxed{\pi}$

ii) $\boxed{\text{Signed area}}$ since $f(x) < 0$ on $(\frac{1}{2}, 1]$.